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OPTIMAL PRODUCTION AND TRADE IN INTERNATIONAL TITANIUM MARKETS

by

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INTRODUCTION AND RATIONALE

The transformation of metallic minerals into metal products entails a variety of processing stages. Moreover, successive stages in the processing chain are often interdependent in the sense that the output of one stage becomes the input to the following one. In a typical metal production process, natural resources are first extracted from the earth and then beneficiated using washing and separation techniques. Afterward, the concentrated ores are smelted and transformed into unwrought metallic forms such as ingots. Later the ingots are melted and used to produce finished metal products.

Despite the input-output relationships linking successive stages in the minerals-to-metals processing chain, sequential stages of production usually need not take place in the same location. Frequently, later processing stages are located in countries with little or none of the natural resources necessary for production of the intermediate or finished metal forms.

Such is the case in the multistage titanium metal industry. Due to its high reactivity with other elements, particularly oxygen, titanium does not exist in nature in its metallic state. Rather, the production of titanium metal typically follows a series of three distinct processing stages. First, ore containing a high concentration of titanium dioxide, such as rutile, is extracted from the earth and beneficiated. In the second stage, the ore is combined with either magnesium or sodium in an electrochemical reduction process. This results in the production of a highly porous, irregularly-shaped metal known as titanium sponge. In the final stage, the titanium sponge is either melted into cast metal products or compacted into ingots far later use in the manufacture of titanium mill products. The conversion of titanium ore to its metallic state requires complex, energy-intensive production technology. The energy necessary to produce one ton of titanium is sixteen times that required far the production of one ton of steel and nearly twice that needed for one ton of aluminum.

The various stages of titanium mineral and metal production are widely dispersed throughout the world. In 1986, Australia, Sierra Leone, and the Republic of South Africa collectively mined over 93 percent of the rutile produced worldwide.[1] However, virtually all of this natural resource was exported to other countries for further processing. During the same period, the three largest producers to titanium sponge and metal, the Soviet Union, the United States, and Japan, possessed little or no high-grade titanium ore.

In its metallic form titanium is light in weight and extremely strong. Throughout its forty-year commercial history, the primary use of titanium metal has been in the manufacture of civilian and military aircraft. However, due to the cyclic nature of the civilian aerospace industry and the unpredictability of military orders for aircraft, future demand for titanium metal has often been difficult to accurately forecast. This has, in turn, led to periods of over capacity and low profitability in the titanium metal industry. Many producers of titanium metal, particularly in the United States, have been reluctant to modernize or expand their production facilities because of the uncertainty of future market conditions.

Due to the importance of titanium metal to the civilian and military aerospace industries, both the ore and metal forms of titanium are classified as strategic materials by the Office of Technology Assessment of

1 U.S. Department of the Interior, Bureau of Mines, *Mineral Ccmmodity Summaries,* (Washington, D.C.: Government Printing Office, 1988), 133.

the U.S. Congress, and are stockpiled by government and industry. The strategic nature of titanium minerals and metal coupled with the geographic dispersion of their production and consumption clearly make this an important industry for economic analysis. Yet despite the strategic significance of this metal, the titanium mineral and metal industries have rarely been the focus of any substantive economic research.

The primary objective of this study, is to determine the optimal production location and trade patterns for the three stages of titanium metal manufacture. By incorporating country-specific data on technology, capacity, production costs, transport costs, tariffs, exchange rates, and demand, into a mixed integer linear programming model, the cost minimizing production levels and trade flows for titanium ore, sponge, and metal are determined.

Economic theory contends that the location of an industry and its concomitant trading patterns are heavily influenced by cost considerations, or in the vernacular of international trade theorists, comparative advantage. Furthermore, industry specialists assert that the future prosperity of the titanium metal industry will largely be determined by its ability to reduce costs and successfully compete with other materials. This is especially true for aerospace applications of titanium metal. Serious substitution threats by composite materials in structural components and advanced ceramics in jet engines must be faced by the titanium metal industry in the near future.[2] Therefore, the determination of the production sites and trade patterns which would minimize costs in the titanium market will serve as an indicator of possible manufacturing strategies which will improve the industry's competitive position.

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2 Ronald F. Balazik and Barry W. Klein, *The Impact of Advanced materials on Conventional Nonfuel Mineral Markets: Selected Forecasts for 1990-2000*, (Washington, D.C.: Government Printing Office, 1987), 6-7

TESTED HYPOTHESIS
AND
THE MIXED INTEGER PROGRAMMING MODEL

The objective of this study is to determine the cost minimizing location of production and patterns of world trade far titanium ore, sponge, and metal which will satisfy a given worldwide level of titanium metal demand. More specifically, the hypothesis that the actual location and level of production at each stage of titanium metal manufacture was no different than the optimal pattern of production is tested. If this hypothesis is accepted it will indicate that the production of titanium minerals and metals has tended to take place in those countries where the combined cost of production and transport was lowest given the geographic distribution and level of titanium metal demand. If this hypothesis is rejected, this would indicate that factors other than production and transport costs have influenced the location of titanium ore and metal production.

To test this hypothesis, the optimal levels of production are compared with the actual quantities of titanium ore, sponge, and metal produced in each country. The Wilcoxon test is then used to determine if the deviation between the two distributions is purely random. If the differences between the actual and optimal spatial distribution of titanium production differ nonrandomly this would indicate that nonprice factors bave significantly influenced worldwide production location and, subsequently, trade in this industry.

To accurately model production and international trade in the multistage titanium industry, a mixed integer linear program is developed for this study. Mixed integer programming refers to a class of programming problems wherein some of the decision variables are restricted to integers. In the present model, binary (0-l) integer variables are used to determine the optimal location of titanium ore, sponge, and metal production from a given set of sites. They also enable one to incorporate fixed production costs into the model.

The objective of the mixed integer program formulated for this study is to determine the level of output at each of the possible titanium ore, sponge, and ingot production sites, along with the pattern of international trade which will minimize the sum of production and transportation costs for a given worldwide spatial distribution of titanium metal demand. The three stages of titanium metal manufacture, ore extraction, sponge production, and metal production, are made interdependent in the model using constraints which link demand for titanium ore and sponge with the materials requirements for a specified quantity of titanium metal.

From a theoretical perspective, this study may be viewed as a fundamental problem of economic choice. Essentially, there exist sets of feasible locations for three economic activities, titanium ore extraction, sponge production, and ingot manufacture, from which an economic choice is made. In order to determine the optimal production distribution for the titanium industry, it is necessary to know the quantity of titanium metal demanded at all consumption points, production costs at all possible production sites, and transportation costs between all feasible production sites and consumption locations. Demand for the final product, in the present case titanium ingot, is taken as exogenous.

Empirically, the problem is essentially a variant of the standard linear programming transportation problem. However, rather than minimizing only transport costs, the objective is to minimize production costs, both fixed and variable, as well as transport costs. The solution to such a model will yield the optimum patterns of production and trade in the multistage titanium industry capable of meeting demand at minimum cost, given existing capacities and costs which vary across producing locations.

The model is short-run in nature, as evidenced by the inclusion of fixed production costs in the objective function, with markets and feasible supply sources specified *a priori.* As a result, the potential for bottlenecks in the system exists with respect to production capacities and the quantities af commodities which may be shipped to each market. The mixed integer program developed for this study is capable of assessing such problems.

MIXED INTEGER PROGRAM[3]

Let i = producing country

j = consuming country

o = set of countries with ore reserves

s = set of countries with sponge production capacity

m = set of countries with metal production capacity

o' = set of ore consuming countries

s' = set of sponge consuming countries

m' = set of metal consuming countries

x^o_{ij} = quantity of ore shipped from country i to country j

x^s_{ij} = quantity of sponge shipped from country i to country j

x^m_{ij} = quantity of metal shipped from country i to country j

y^o_i = 0-1 decision variable for ore production in country i, where y^o_i will equal one if ore production takes place in country i, otherwise y^o_i will equal zero.

y^s_i = 0-1 decision variable for sponge production in country i, where y^s_i will equal one if sponge production takes place in country i, otherwise y^s_i will equal zero.

y^m_i = 0-1 decision variable for metal production in country i, where y^m_i will equal one if sponge production takes place in country i, otherwise y^m_i will equal zero.

The objective of the mixed integer program is to determine the optimal values of the decision variables, x^o_{ij}, x^s_{ij}, x^m_{ij}, y^o_i, y^s_i, y^m_i, which minimize K, total production and transportation costs for the multistage titanium industry as follows:

3 The assumptions employed in this model are outlined in Appendix A.

$$
\begin{aligned}
k = & \sum_{i\in o}\sum_{j\in o} c_i^o x_{ij}^o + \sum_{i\in o} f_i^o y_i^o + \sum_{i\in o}\sum_{j\in o'} t_{ij}^o x_{ij}^o \\
& + \sum_{i\in o}\sum_{j\in o'} r_{ij}^o x_{ij}^o + \sum_{i\in s}\sum_{j\in s'} c_i^s x_{ij}^s + \sum_{i\in s} f_i^s y_i^s \\
& + \sum_{i\in s}\sum_{j\in s'} t_{ij}^s x_{ij}^s + \sum_{i\in s}\sum_{j\in s'} r_{ij}^s x_{ij}^s \\
& + \sum_{i\in m}\sum_{j\in m'} c_i^m x_{ij}^m + \sum_{i\in m} f_i^m y_i^m + \sum_{i\in m}\sum_{j\in m'} t_{ij}^m x_{ij}^m \\
& + \sum_{i\in m}\sum_{j\in m'} r_{ij}^m x_{ij}^m
\end{aligned} \qquad (1)
$$

subject to

$$\sum_{j\in o'} x_{ij}^o \leqslant U_i^o y_i^o \qquad \text{for all } i\in o \qquad (2)$$

$$\sum_{j\in s'} x_{ij}^s \leqslant U_i^s y_i^s \qquad \text{for all } i\in s \qquad (3)$$

$$\sum_{j\in m'} x_{ij}^m \leqslant U_i^m y_i^m \qquad \text{for all } i\in m \qquad (4)$$

$$\sum_{i\in o} x_{ij}^o \geqslant a^{o,s} \sum_{j\in s'} x_{ij}^s \qquad \text{for all } i\in s,\ \text{for all } j\in o',\ \text{where } s=o' \qquad (5)$$

$$\sum_{i\in s} x_{ij}^s \geqslant a^{s,m} \sum_{j\in m'} x_{ij}^m \qquad \text{for all } i\in m,\ \text{for all } j\in s',\ \text{where } m=s' \qquad (6)$$

$$\sum_{i\in m} x_{ij}^m \geqslant D_j^m \qquad \text{for all } j\in m' \qquad (7)$$

$$y_i^o = \begin{bmatrix} 1 & \text{if } x_{ij}^o > 0 \\ 0 & \text{otherwise} \end{bmatrix} \qquad \text{for all } i\in o \qquad (8)$$

$$y_i^s = \begin{bmatrix} 1 & \text{if } x_{ij}^s > 0 \\ 0 & \text{otherwise} \end{bmatrix} \qquad \text{for all } i\in s \qquad (9)$$

$$y_i^m = \begin{bmatrix} 1 & \text{if } x_{ij}^m > 0 \\ 0 & \text{otherwise} \end{bmatrix} \qquad \text{for all } i\in m \qquad (10)$$

$$x_{ij}^o \geqslant 0 \qquad \text{for all } i\in o,\ \text{for all } j\in o' \qquad (11)$$

$$x_{ij}^s \geqslant 0 \qquad \text{for all } i\in s,\ \text{for all } j\in s' \qquad (12)$$

$x^m_{ij} \geqslant 0$ for all $i \in m$, for all $j \in m'$ (13)

A concise explanation of each of the elements in aforementioned mixed integer program io provided in the following sections. For the convenience of the reader, the objective function and each of the constraints in the model is repeated as it io discussed.

1. Objective Function

The objective of this study is to determine the level of output at each of the possible titanium production sites and the pattern of international trade which will minimize the sum of production and transport costs while fulfilling titanium demand worldwide. This is achieved by the determination of the values of the decision variables, x^o_{ij}, x^s_{ij}, x^m_{ij}, which represent, respectively, flows of titanium ore, sponge, and metal between supplying country i and demanding country j, and the 0-1 integer decision variables for ore, sponge, and metal production, y^o_i, y^s_i, y^m_i, which minimize the value of the objective function.

The objective function employed in this analysis measures variable production costs ($c^o_j x^o_{ij}$, $c^s_i x^s_{ij}$, $c^m_i x^m_{ij}$), fixed production costs ($f^o_i y^o_i$, $f^s_i y^s_i$, $f^m_i y^m_i$), transport costs ($t^o_{ij} x^o_{ij}$, $t^s_{ij} x^s_{ij}$, $t^m_{ij} x^m_{ij}$), and tariffs ($r^o_{ij} x^o_{ij}$, $r^s_{ij} x^s_{ij}$, $r^m_{ij} x^m_{ij}$), for each stage of titanium production and distribution. The superscripts, o, s, m, denote the stage of titanium production, either ore, sponge, or metal, respectively. These various costs are summed over the sets of countries with titanium ore reserves ($i \in o$), titanium sponge production capacity ($i \in s$), and titanium metal production capacity ($i \in m$), as well as the sets of countries consuming titanium ore ($j \in o'$), sponge ($j \in s'$), and metal ($j \in m'$). Mathematically, the objective of the mixed integer program is stated as follows:

$$
\begin{aligned}
\text{Min } k = & \sum_{i\in o}\sum_{j\in o} c_i^o x_{ij}^o + \sum_{i\in o} f_i^o y_i^o + \sum_{i\in o}\sum_{j\in o'} t_{ij}^o x_{ij}^o \\
& + \sum_{i\in o}\sum_{j\in o'} r_{ij}^o x_{ij}^o + \sum_{i\in s}\sum_{j\in s'} c_i^s x_{ij}^s + \sum_{i\in s} f_i^s y_i^s \\
& + \sum_{i\in s}\sum_{j\in s'} t_{ij}^s x_{ij}^s + \sum_{i\in s}\sum_{j\in s'} r_{ij}^s x_{ij}^s \\
& + \sum_{i\in m}\sum_{j\in m'} c_i^m x_{ij}^m + \sum_{i\in m} f_i^m y_i^m + \sum_{i\in m}\sum_{j\in m'} t_{ij}^m x_{ij}^m \\
& + \sum_{i\in m}\sum_{j\in m'} r_{ij}^m x_{ij}^m
\end{aligned}
$$

2. Ore production Constraint

If country i which possesses titanium ore reserves, ($i\in o$), is selected for ore production, then output must not exceed the country's ore reserves. Furthermore, ore production in country i will be zero if y^o_i, the 0-l integer decision variable associated with ore production in that country, is zero.

$$\sum_{j\in o'} x_{ij}^o \leqslant U_i^o y_i^o \qquad \text{for all } i\in o \qquad (2)$$

3. Sponge Production Constraint

If country i which possesses titanium sponge production capacity, ($i\in s$), is selected for sponge manufacture, then output must not exceed the country's sponge production capacity. In addition, sponge production in country i is zero if y^s_i, the 0-l integer decision variable associated with sponge production in that country, is zero.

$$\sum_{j\in s'} x_{ij}^s \leqslant U_i^s y_i^s \qquad \text{for all } i\in s \qquad (3)$$

4. Metal production Constraint

If country i with titanium metal production capacity, ($i\in m$), is selected for metal production, then output must not exceed the country's metal production capacity. Furthermore, metal production is zero in country i if y^m_i, the 0-l integer decision variable representing titanium metal production in that country, is zero.

$$\sum_{j\in m'} x_{ij}^{m} \leqslant U_i^{m} y_i^{m} \qquad \text{for all } i\in m \qquad (4)$$

5. Ore Demand/Materials Balance Constraint

This constraint combines the materials balance relationship existing between the titanium ore and sponge production with titanium ore demand. The input-output coefficient, $a^{o,s}$, measures the quantity of ore needed to produce one unit of titanium sponge. If a country produces titanium sponge then shipments of ore to the country must be sufficient for its level of sponge production. Clearly, if a country is to produce sponge, then it must demand ore. Hence, the set of countries producing sponge, s, must be equal to the set of countries demanding ore, o'.

$$\sum_{i\in o} x_{ij}^{o} \geqslant a^{o,s} \sum_{j\in s'} x_{ij}^{s} \qquad \begin{array}{l} \text{for all } i\in s \\ \text{for all } j\in o' \\ \text{where } s=o' \end{array} \qquad (5)$$

6. Sponge Demand/Materials Balance Constraint

This constraint combines the materiale balance relationship existing between titanium sponge and metal production with titanium sponge demand. The input-output coefficient, $a^{s,m}$, measures the quantity of titanium sponge needed to produce one unit of titanium metal. If a country produces titanium metal then shipments of titanium sponge to that country must be sufficient for its level of metal production. Clearly, if a country is to produce titanium metal then it must demand titanium sponge. Hence, the set of countries producing metal, m, must be equal to the set of countries demanding sponge, s'.

$$\sum_{i\in s} x_{ij} \geqslant a \sum_{j\in m'} x_{ij} \qquad \begin{array}{l} \text{for all } i\in m \\ \text{for all } j\in s' \\ \text{where } m=s' \end{array} \qquad (6)$$

7. Metal Demand Constraint

Demand for titanium metal is assumed to be exogenous. This assumption from the fact that the present model is short run in nature. The following constraint ensures that the total amount of titanium metal shipped from all of the i supplying countries to the j^{th} metal demanding country is sufficient for the level of metal demand in country j.

$$\sum_{i \in m} x^m_{ij} \geq D^m_j \qquad \text{for all } j \in m' \qquad (7)$$

8. Binary Integer Constraint for Ore Production

The binary integer variable y^o_i represents the decision to produce or not produce ore in country i, where i is an element of the set of countries with titanium ore reserves, ($i \in o$). If a country is selected far ore production, then y^o_i is equal to one. However, if a country io not selected for ore production, then y^o_i is equal to zero.

$$y^o_i = \begin{bmatrix} 1 \text{ if } x^o_{ij} > 0 \\ 0 \text{ otherwise} \end{bmatrix} \qquad \text{for all } i \in o \qquad (8)$$

9. Binary Integer Constraint for Sponge Production

The binary integer variable y^s_i represents the decision to produce or not produce sponge in country i, where i is an element of the set of countries with sponge production capacity, ($i \in s$). If a country is selected for sponge production, then y^s_i is equal to one. However, if a country is not selected for sponge production, then y^s_i is equal to zero.

$$y^s_i = \begin{bmatrix} 1 \text{ if } x^s_{ij} > 0 \\ 0 \text{ otherwise} \end{bmatrix} \qquad \text{for all } i \in s \qquad (9)$$

10. Binary Integer Constraint for Metal Production

The binary integer variable y^m_i represents the decision to produce or not produce titanium metal in country i, where i is an element of the set of countries with metal production capacity, ($i \in m$). If a country is selected for metal production, then y^m_i is equal to one. However, if a country is not selected for metal production, then y^m_i is equal to zero.

$$y_i^m = \begin{bmatrix} 1 & \text{if } x_{ij}^m > 0 \\ 0 & \text{otherwise} \end{bmatrix} \qquad \text{for all } i \in s \qquad (10)$$

11. Nonnegativity Constraints

All shipments of commodities in the model between supplying and demanding countries must be nonnegative. Constraints (11), (12), and (13), ensure, respectively, that shipments of titanium ore, sponge, and metal in the model are either positive or equal to zero.

$$x_{ij}^o \geqslant 0 \qquad \text{for all } i \in o, \quad \text{for all } j \in o' \qquad (11)$$

$$x_{ij}^s \geqslant 0 \qquad \text{for all } i \in s, \quad \text{for all } j \in s' \qquad (12)$$

$$x_{ij}^m \geqslant 0 \qquad \text{for all } i \in m, \quad \text{for all } j \in m' \qquad (13)$$

EMPIRICAL RESULTS AND CONCLUSIONS

The objective of this study is the determination of the optimal worldwide spatial distribution of production and subsequent trade patterns for the multistage titanium industry. To accomplish this task, a mixed integer linear programming model was developed to estimate the cost minimizing location of titanium ore, sponge, and ingot production. Afterward, the results obtained with the program were used to test the hypothesis that the actual location and level of production at each stage of titanium metal manufacture was no different than the optimal pattern of production in 1984.

The estimated cost minimizing patterns of production and trade in titanium minerals in 1984 are illustrated in tables 1 and 2. For the first stage of titanium metal manufacture, specifically, that of ore extraction and beneficiation, the optimal solution discloses that only one country would have engaged in titanium ore production in 1984. Specifically, Australia would have supplied all the rutile and ilmenite necessary for titanium metal manufacture. Such a result is reasonable in light of the high capital costs associated with titanium ore extraction and beneficiation, as

TABLE 1

OPTIMAL (COST MINIMIZING) SHIPMENTS OF RUTILE, 1984

(Data in short tons)

FROM \ TO	E. U.S.A.	W. U.S.A.	JAPAN	U.K.	TOTAL SUPPLY
E. AUSTRALIA	20900.0	52800.0	76859.2	12100.0	162659.2
E. U.S.A.	0.0	0.0	0.0	0.0	0.0
INDIA	0.0	0.0	0.0	0.0	0.0
REP. OF SOUTH AFRICA	0.0	0.0	0.0	0.0	0.0
U.S.S.R.	0.0	0.0	0.0	0.0	0.0
SIERRA LEONE	0.0	0.0	0.0	0.0	0.0

TABLE 2

OPTIMAL (COST MINIMIZING) SHIPMENTS OF ILMENITE, 1984

(Data in short tons)

FROM \ TO	U.S.S.R	PEOPLES REP. OF CHINA	TOTAL SUPPLY
W. AUSTRALIA	130000.0	7500.0	137500.0
U.S.S.R	0.0	0.0	0.0
PEOPLES REP. OF CHINA	0.0	0.0	0.0

well as the vast economic reserves of these minerals present alonq the eastern and western coasts of Australia. In addition, no tariffs were levied upon Australian rutile imports by any of the rutile consuminq countries in the model. Similarly, for those few countries which did impose tariffs on ilmenite imports, the duty was extremely low. Furthermore, the relatively low transport costs associated with these minerals effectively eliminates the locational advantage held by other nations endowed with titanium ores yet in closer proximity to titanium mineral consumption points.

In interpreting the aforementioned results concerning optimal titanium ore production and trade, it is important to recall that the model developed for the present study places primary emphasis upon titanium metal markets. Pigment demand for titanium minerale was not included in the model due to the dissimilarity of this commodity to titanium metal. Furthermore, since only ten percent of the titanium minerals consumed worldwide are used to produce titanium metal, the probability that the present model underestimates production and trade in rutile and ilmenite must be given proper consideration.

For the second stage of titanium metal manufacture, the cost minimizing model solution predicts that titanium sponge production would have taken place in all five countries that possessed titanium sponge production capacity in 1984. By far the greatest level of titanium sponge production, 52,000 short tons, would have taken place in the Soviet Union, while Japan and the United States would have also been significant producers of the metal under the cost minimization criterion. The optimal spatial distribution of titanium sponge production and subsequent trade patterns for 1984 are shown in table 3.

In addition, the model's results indicate that international trade in titanium sponge would chiefly be conducted by the Japanese. It was estimated that Japan would ship significant quantities of titanium sponge to the Soviet Union, West Germany, and the United States. Moreover, under the optimal, cost minimizing solution, Japan would be the sole foreign supplier of titanium sponge to the United States. Such a result is, in fact, quite realistic given the fact that between 1983 and 1986, 95 percent of U.S.

TABLE 3

OPTIMAL (COST MINIMIZING) SHIPMENTS OF TITANIUM SPONGE, 1984

(Data in short tons)

FROM \ TO	E. U.S.A.	W. U.S.A.	JAPAN	U.K.	U.S.S.R.	W. GERMANY	TOTAL SUPPLY
E. U.S.A.	0.0	9500.0	0.0	0.0	0.0	0.0	9500.0
W. U.S.A.	0.0	24000.0	0.0	0.0	0.0	0.0	24000.0
JAPAN	0.0	1936.0	14000.0	0.0	15000.0	4000.0	34936.0
U.K.	0.0	0.0	0.0	5000.0	0.0	500.0	5500.0
U.S.S.R.	0.0	0.0	0.0	0.0	52000.0	0.0	52000.0
PEOPLES REP. OF CHINA	0.0	0.0	0.0	0.0	3000.0	0.0	3000.0

titanium sponge importa came from Japan.[4]

In general, those countries which engaged in titanium sponge production also tended to be major consumers of the metal. Therefore, demand for titanium sponge was typically met by domestic producers, and any excess supply was exported. Given the higher transport costs associated with the shipment of titanium sponge as well as the fact that tariffs on titanium sponge reached as high as 25 percent *ad valorem* for some countries included in the model,[5] the aforementioned model results are quite sound.

In the final stage of titanium metal manufacture, the estimated cost minimizing location of ingot production and subsequent trade flows were found to be similar to those estimated for titanium sponge production. In the optimal solution, titanium ingot manufacture would take place in all of the countries possessing ingot production capacity in 1984. The highest level of output was estimated to take place in the Soviet Union. As in the case of titanium sponge trade, demand for titanium ingot in those countries which were also producers of the metal tended to be met by domestic firms. The optimal location of production in titanium ingot and patterns of international trade in 1984 are shown in table 4.

Following the determination of the optimal production and trade patterns for the multistage titanium industry, the data generated by the mixed integer program solution were used to test the hypothesis that the optimal spatial distribution of production at each stage of processing was not different from the actual distribution in 1984. The Wilcoxon Matched-Pairs Signed-Ranks Test was used to analyze the differences between the actual and optimal production levels at each stage of titanium ore and metal processing. This nonparametric statistical test takes into consideration both the direction and the magnitude of the differences between pairs of data.[6]

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4 U.S. Department of the Interior, Bureau of Mines, *Mineral Commodity Summaries,* (Washington, D.C.: U.S. Government Printing Office, 1988), 170.

5 International Customs Tariffs Bureau, *International* Customs *Journal,* (Brussels, Belgium: International Customs Tariffs Bureau).

6 See, for example, Sidney Siegel, *Nonparametric Statistics for the Behavioral Sciences,* (New York: McGraw-Hill Book Company, Inc., 1957), 77-83.

TABLE 4

OPTIMAL (COST MINIMIZING) SHIPMENTS OF TITANIUM INGOT, 1984

(Data in short tons)

TO / FROM	E. U.S.A.	W. U.S.A.	JAPAN	U.K.	W. GERMANY	U.S.S.R.	FRANCE	ITALY	CANADA	TOTAL SUPPLY
E. U.S.A.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
W. U.S.A.	17718.0	17718.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	35436.0
JAPAN	0.0	0.0	14000.0	0.0	0.0	0.0	0.0	0.0	0.0	14000.0
U.K.	0.0	0.0	0.0	0.0	0.0	0.0	500.0	4500.0	0.0	5000.0
U.S.S.R.	0.0	0.0	1000.0	2500.0	7500.0	47500.0	0.0	4000.0	7500.0	70000.0
W. GERMANY	0.0	0.0	0.0	0.0	0.0	0.0	4500.0	0.0	0.0	4500.0

The actual and optimal levels of rutile, ilmenite, titanium sponge, and titanium ingot production for 1984 ara presented in tables 5 through 8, respectively. After conducting the Wilcoxon Test on each of the aforementioned titanium commodities, it was found that the null hypothesis (the actual location and level of production was not different from the optimal distribution) could be rejected for the titanium ore and sponge stages of production.

Given the results of this test, it can be concluded that for the ore and sponge stages of titanium metal manufacture, factors other than production and distribution costs influenced production decisions for these titanium commodities. Without inside industry information, it is difficult to precisely pinpoint the specific nonprice factors which influenced managers production decisions at these stages of titanium production. However, in the case of ore production, the results obtained with the Wilcoxon test must be tempered by the fact that pigment demand for titanium ores was excluded from the present model. Therefore, the estimates derived for titanium ore production and demand with this model are substantially lower than actual figures. In the case of titanium sponge production, the deviation between the optimal and actual production choices may possibly be the result of the risk aversion on the part of titanium sponge consumers who are willing to pay premium prices in order to maintain secure supplies of the metal.

At the final stage of titanium metal manufacture, ingot production, the null hypothesis was accepted. This would indicate that the actual spatial location of titanium ingot production did not differ significantly from the cost minimizing distribution. It can, therefore, be concluded that this stage of titanium metal manufacture tended to locate at those sites where the combined costs of production and distribution were minimized.

TABLE 5

ACTUAL AND OPTIMAL (COST MINIMIZING) PRODUCTION OF

RUTILE BY COUNTRY, 1984

(Data in short tons)

Country	Actual[1] Production	Optimal Production
U.S.A.	W	0.0
Australia	200000.0	162659.2
India	8000.0	0.0
Sierra Leone	101000.0	0.0
Rep. of South Africa	62000.0	0.0
Sri Lanka	9000.0	0.0
U.S.S.R.	11000.0	0.0

Note: W denotes data withheld to avoid disclosing proprietary company data.

1 Source: U.S. Department of the Interior, Bureau of Mines, *Mineral Commodity Summaries,* (Washington, D.C.: U.S. Government Printing Office, 1986), 133.

TABLE 6

ACTUAL AND OPTIMAL (COST MINIMIZING) PRODUCTION OF

ILMENITE BY COUNTRY, 1984

(Data in short tons)

Country	Actual[1] Production	Optimal Production
U.S.A.	W	0.0
Australia	1210000.0	137500.0
Brazil	55000.0	0.0
Finland	184000.0	0.0
India	165000.0	0.0
Malaysia	215000.0	0.0
Norway	606000.0	0.0
Sri Lanka	88000.0	0.0
China	154000.0	0.0
U.S.S.R.	485000.0	0.0

Note: W denotes data withheld to avoid disclosing proprietary company data.

1 Source: U.S. Department of the Interior, Bureau of Mines, *Mineral Commodity Summaries,* (Washington, D.C.: U.S. Government Printing Office, 1986), 73.

TABLE 7

ACTUAL AND OPTIMAL (COST MINIMIZING) PRODUCTION OF

TITANIUM SPONGE BY COUNTRY, 1984

(Data in short tons)

Country	Actual[1] Production	Optimal Production
U.S.A.	24326.0	33500.0
Japan	16938.0	34936.0
U.K.	2500.0	5500.0
U.S.S.R.	46000.0	52000.0
Peoples Rep. of China	2000.0	3000.0

1 Source: U.S. Department of the Interior, Bureau of Mines, *Mineral Commodity Summaries*, (Washington, D.C.: U.S. Government Printing Office, 1986), 169.

TABLE 8

ACTUAL AND OPTIMAL (COST MINIMIZING) PRODUCTION OF

TITANIUM INGOT BY COUNTRY, 1984

(Data in short tons)

Country	Actual[1] Production	Optimal Production
U.S.A.	32840.1	35436.0
Japan	18631.8	14000.0
U.K.	3375.0	5000.0
U.S.S.R.	62100.0	70000.0
West Germany	3375.0	4500.0

1 Actual data unavailable. Estimates based on 1.35 times actual sponge production in 1984.

APPENDIX A

ASSUMPTIONS

In formulating the mixed integer program used in this study, a number of assumptions were made. These assumptions may be classified into one of two groups: (a) assumptions which are common to the methods of linear programming, or, (b) assumptions which are unique to the titanium production and trade model developed for this study. In the following sections, each of the aasumptions is stated and any underlying economic implications associated with an assumption are discussed.

STANDARD LINEAR PROGRAMMING ASSUMPTIONS

The following are assumptions typically made in linear programming problems.[7]

1. Certainty: It is assumed that all constants contained in the objective function and the constraints are known with certainty.

2. Nonnegativity: Only nonnegative activity levels are feasible. Therefore, the non-integer choice variables in the model, x^{o}_{ij}, x^{s}_{ij}, x^{m}_{ij}, which respectively represent flows of titanium ore, sponge, and ingot from producing country i to consuming country j, can take on only positive values or a value of zero.

3. Linear Objective Function: It is assumed that the objective function is expressed in terms of a linear function. This assumption is consistent with the underlying Leontief production technology employed in the model. More specifically, given the Leontief production function

$$f(x_1, x_2) = \min(ax_1, bx_2)$$

where x_1 and x_2 are inputs, and a and b represent the strictly positive

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7 Narendra Paul Loomba and Ephraim Turban, *Applied Programming for Management,* (New York: Holt, Rinehart, and Winston, 1974), 56-57.

input intensities of factors x_1 and x_2, respectively, the corresponding dual cost function is of the form

$$c(w_1, w_2, y) = w_1 y/a + w_2 y/b = (w_1/a + w_2/b)y$$

where w_1 and w_2 represent input prices of factors x_1 and x_2 respectively, y is the level of output, and a and b respectively measure the strictly positive input intensities of factors x_1 and x_2.

4. Linear Constraints: The constraints employed in the problem are expressed in terms of linear inequalities.

5. Proportionality: The linearity of the objective function and each of the constraints in the model results in proportionality in the system. This implies: (a) the contribution to the objective function of each activity is directly proportional to the level of that activity, and (b) the use of a factor by each activity is directly proportional to the level of that activity. The constants of proportionality in the constraints, therefore, indicate constant returns to scale in the system, which is, in turn, consistent with the underlying Leontief production technology.

6. Additivity: The total of all activities is equal to the sum of each individual activity. Therefore, the total cost incurred by a series of activities, in the present case, titanium ore, sponge, and metal production, would be equal to the summation of the cost of each of the aforementioned individuai activities. Also, the assumption of additivity would imply that the total utilization of a resource is equal to the sum of the resource's use by each of the individual activities.

7. Maximization (or Minimization) of a Single Goal: It is assumed that a single goal can be identified as the objective of the linear programming problem. In the model developed for this study, minimization of the costs incurred in the production and distribution

of titanium ore, sponge, and ingot is specified as the singular goal of the mixed integer program.

8. Divisibility: With the exception of the binary (0-l) variables identified in the program, y^o_i, y^s_i, y^m_i, which indicate whether production is taking place in location i, it is assumed that the other choice variables in the model, x^o_{ij}, x^s_{ij}, x^m_{ij}, which represent flows of commodities from producing country i to consuming country j, are continuous.

ASSUMPTIONS UNIQUE TO THE TITANIUM PRODUCTION AND TRADE MODEL

In addition to the standard linear programming assumptions cited in the previous section, the following assumptions are also used in formulating the titanium production and trade model.

1. The location and the magnitude of titanium ore reserves, sponge production capacity, and metal production capacity are assumed exogenous in the model.

2. The cost function at each stage of production is based on a Leontief production technology with input intensities fully specified *a priori.* However, in an effort to accurately model production in the international titanium industry, the cost function differs slightly in centrally planned economies engaging in titanium sponge production. This is due to the fact that titanium sponge producers in China and the soviet Union employ ilmenite as the natural resource input, whereas in all other titanium sponge producing countries, rutile is used as the mineral feedstock. By making this distinction in the model's cost function, it is felt more precise empirical results will be obtained.

3. The input-output coefficients linking ore demand and sponge

production, $a^{o,s}$, and sponge demand and metal production, $a^{s,m}$, are assumed to be identical across producing countries.

4. Demand for titanium metal in each consuming country is exogenous, while demands for titanium ore and sponge are endogenous.

5. Input prices in countries with titanium reserves or production capacity are assumed exogenous.

6. Transport costs are assumed to be a linear function of the distance between supplying and demanding countries.

7. There is no transshipment of titanium commodities in the model. This is consistent with the triangular inequality which states

$$t_{ij} \leqslant t_{ik} + t_{kj}$$

where t_{ij} is the united transportation cost for a commodity shipped from site i to site j, t_{jk} is the unit transportation cost for a commodity shipped from site i to site k, and t_{kj} is the unit transportation cost for a commodity shipped from site k to site j.

8. Intracountry transport costs are assumed to be zero. This assumption effectively collapses all countries in the model into production or consumption points and thereby implies a punctual approach to location theory. This approach to location theory is closely tied to the doctrine of comparative advantage, in so much as the pattern of production location across multiple geographic points is based upon predetermined relative resource endowments and received prices at the points.

9. Tariffs on all titanium commodities are assumed to be exogenous.